Fluid Mechanics
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Lecture No. – 09
Conservation of Momentum

Welcome all of you to fluid mechanics course. Today, I am going to deliver lecture on conservation of momentum. As you know, in the last class we discussed about conservation of mass. Also, we have solved few problems based on the conservation of mass.

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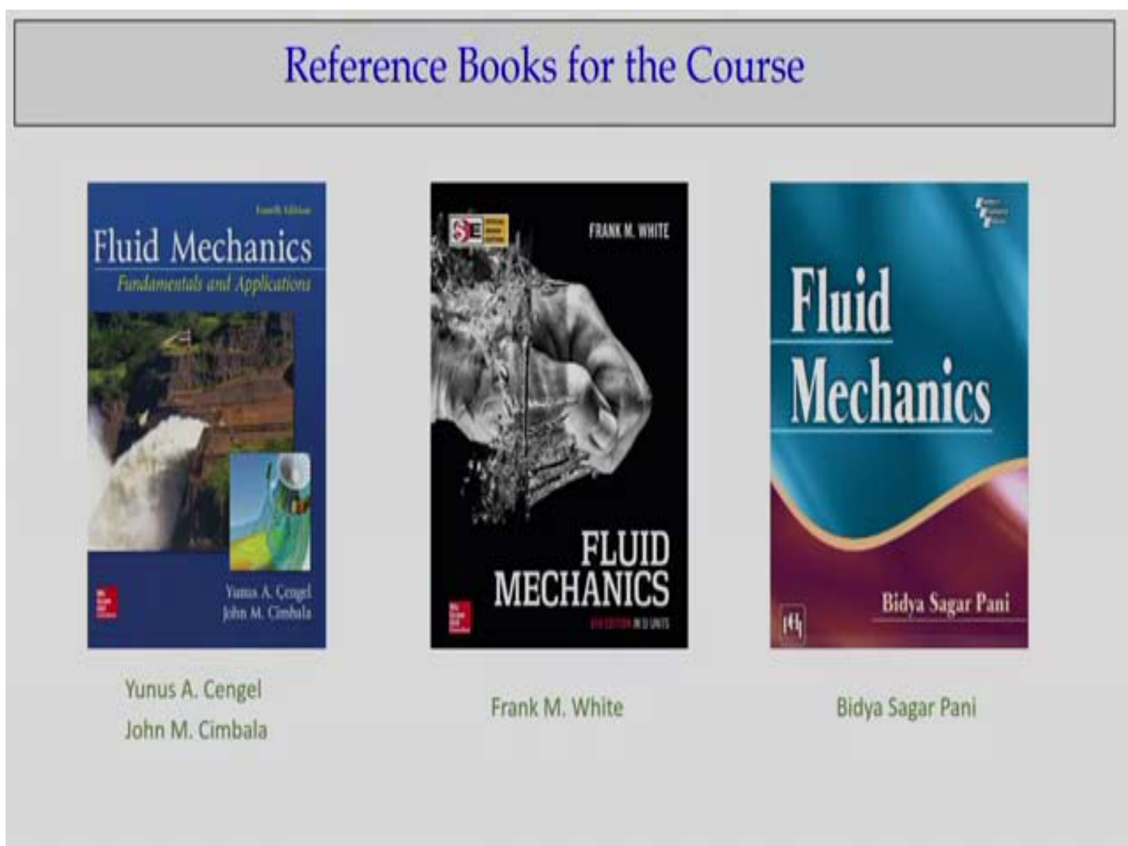

Basically, we have been following the Reynolds transport theorem as a basic concept to apply the setup of the system into physical equations to the control volume level. And, then, at the control column level, we have approximation of extensive and intensive properties and as of now we have derived mass conservation equations. Today, I am going to derive conservation of linear momentum.

Again, I can tell you, book wise, the Cengel and Cimbala, The Fluid Mechanics Fundamentals and Applications. This book has given very clearly the illustrations of this concept of Reynolds transport theorem, the conservation of mass, conservation of linear momentum, angular momentum, and the energy concept which is more descriptive type and that is why it gives enough to a student to understand this concept.

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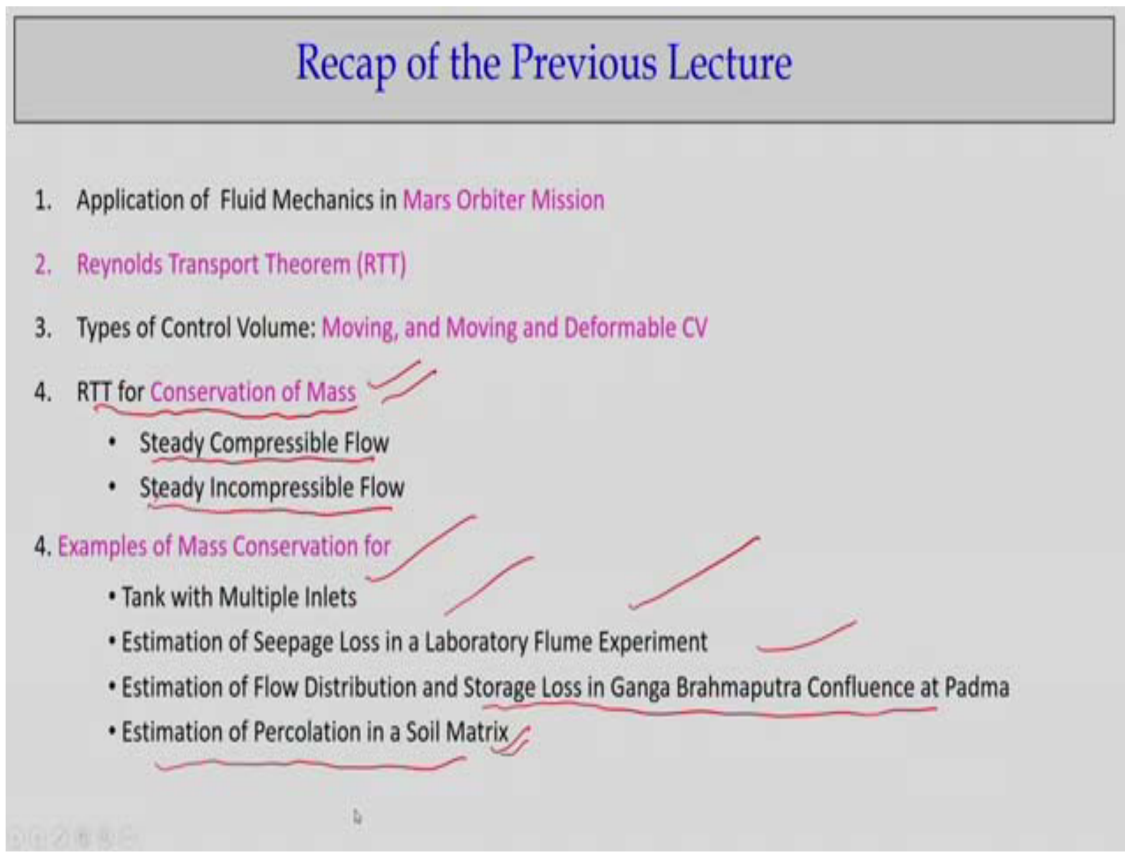

So, now let us come back to the last class what we studied. As I told you, we discussed about the Reynolds transport theorem for conservation of mass and when we apply this conservation of mass to the Reynolds transport theorem, we have two basic assumptions, that is, with respect to time is it a steady or unsteady. So, the steady we do the approximations of many fluid flow problems which are steady problems.

Then, with respect to density change or the variations of the density, we divide it, flow is compressible or incompressible. So, we can have two types of approximations, steady compressible, steady incompressible. So, when you have the steady assumptions, you can remember that the component of Reynolds transport theorem of time, differentiate components become 0 or the volume integral component part of the Reynolds transport theorem becomes 0. So, it becomes a very easy problem.

You have only this surface integral component and it is equal to 0. So, that is a very simplified case. And when the density is a constant, that means, what happens is densities comes out from the equations which makes us only the scalar product between velocity and the normal vectors, that is what is a scalar quantity. We do surface integrals with respect to area.

So, thus, the problems becomes too simple as compared to if you have compressible flow. So, when you have a steady incompressible flow, most of the case what we consider for flow devices or engineering applications, we can consider steady incompressible flow, then the problems becomes very simplified when you apply for Reynolds transport theorem. As you remember, there are starting with tanks with multiple inlets, the estimation of seepage losses in laboratory flumes. Today, I will repeat the problem, how to do these things.

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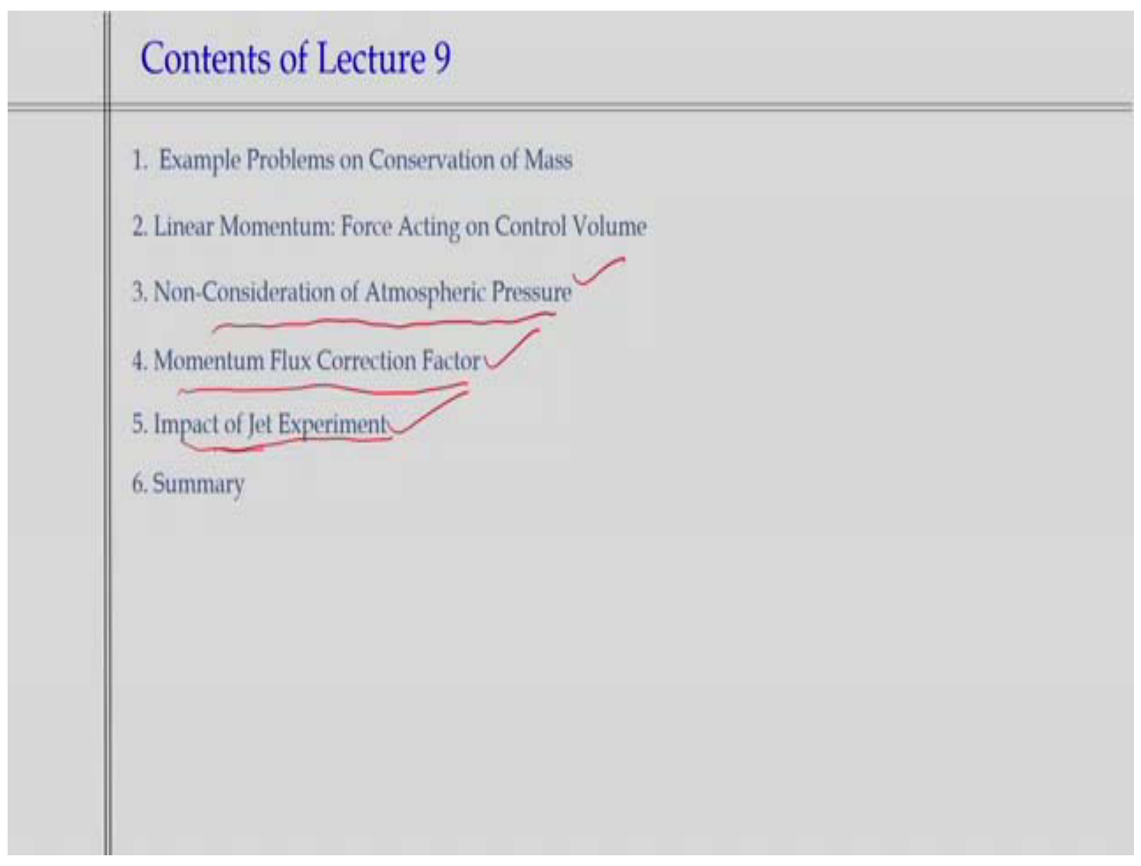

Let me come back to today's lectures, what I will cover. Again, I will give you a few examples on conservation of mass. Then, we will go to write the linear momentum equations for fixed control volumes or moving control volumes. Then, what are the simplifications that need to be done before applying linear momentum equations. That is what we will discuss in terms of non-consideration of atmospheric pressures.

Then, what is called the momentum flux correction factor, how we use it, that is what I will discuss. Then, I will show the impact of jet experiment, then, I will summarise it.

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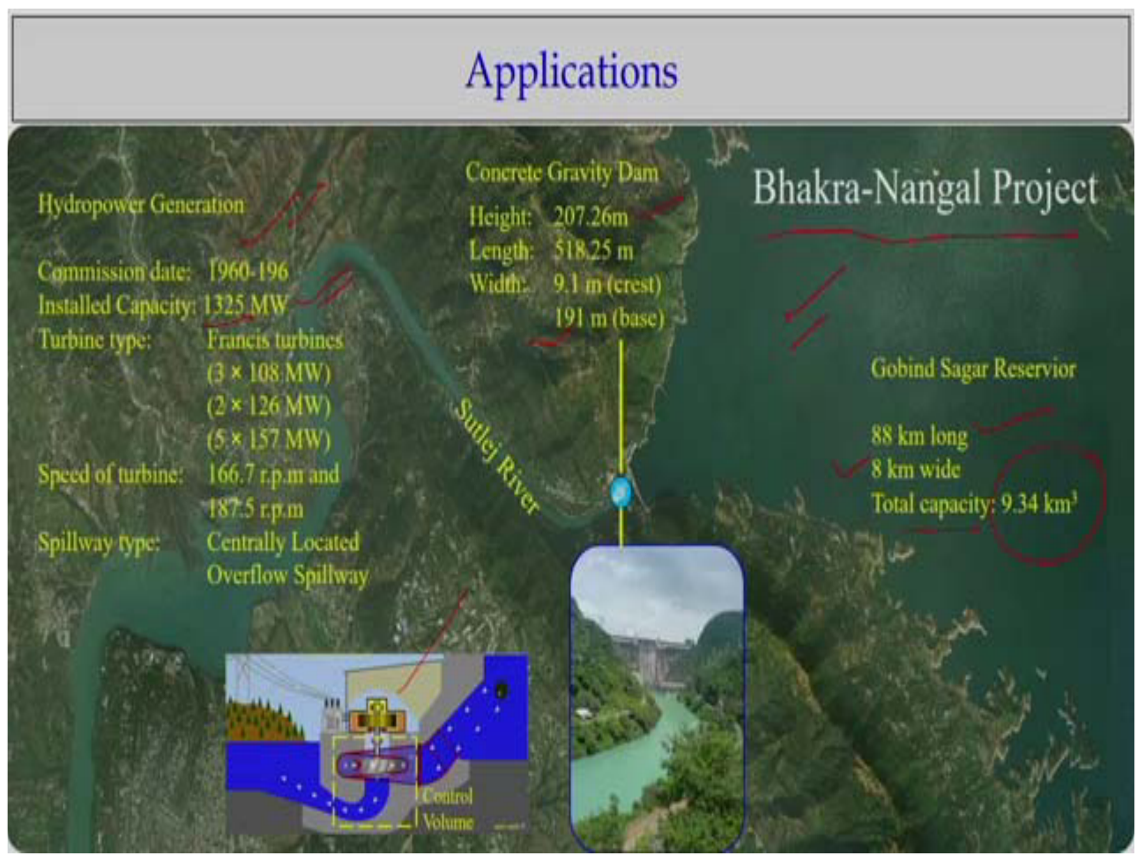

So, before going to these things, you could have heard of this hydro projects, one is one of the largest projects in our country, which is Bhakra Nangal project. If we look at this Bhakra Nangal project, it has a reservoir which is about 88 kilometers long and 8 kilometers wide. And total water storage capacity is about 9.34 kilometer cube, so, huge amount of water storage if you can see in this Google earth imagery.

The dam is located here which is a concrete dam and having a height of 207 meters approximately, and the length is 500 meters, and width varies from at the top 9 meters, as it goes down the base becomes wider and wider which will be 191 meters. So, what I am to say is that, if you look at this project which was initiated or commissioned early in 1950s and 60s, generating and installing hydro power projects about 1300 megawatt power.

So, what you are looking is basic fluid mechanics knowledge. That is what is used to design this hydropower project. So, the basic fluid mechanics what we have that is what is used way back in 1950s to design this Bhakra Nangal project which is one of the successful projects in our country. So, if you look at this way, we will take a lot of hydro power projects and we will tell how to estimate the power potentials, how to estimate what could be the turbine speed, all we can do it.

It is not a difficult task if you have knowledge of fluid mechanics. So, only the knowledge of fluid mechanics and civil engineering excellence is helping us to generate the power at the order of 1300 megawatt powers without polluting the environment. So, the hydropower projects they have the strength. Also, some disadvantages are there, but they are the projects that are implemented and those project components we can understand if we understand fluid mechanics well. That is my point for you.

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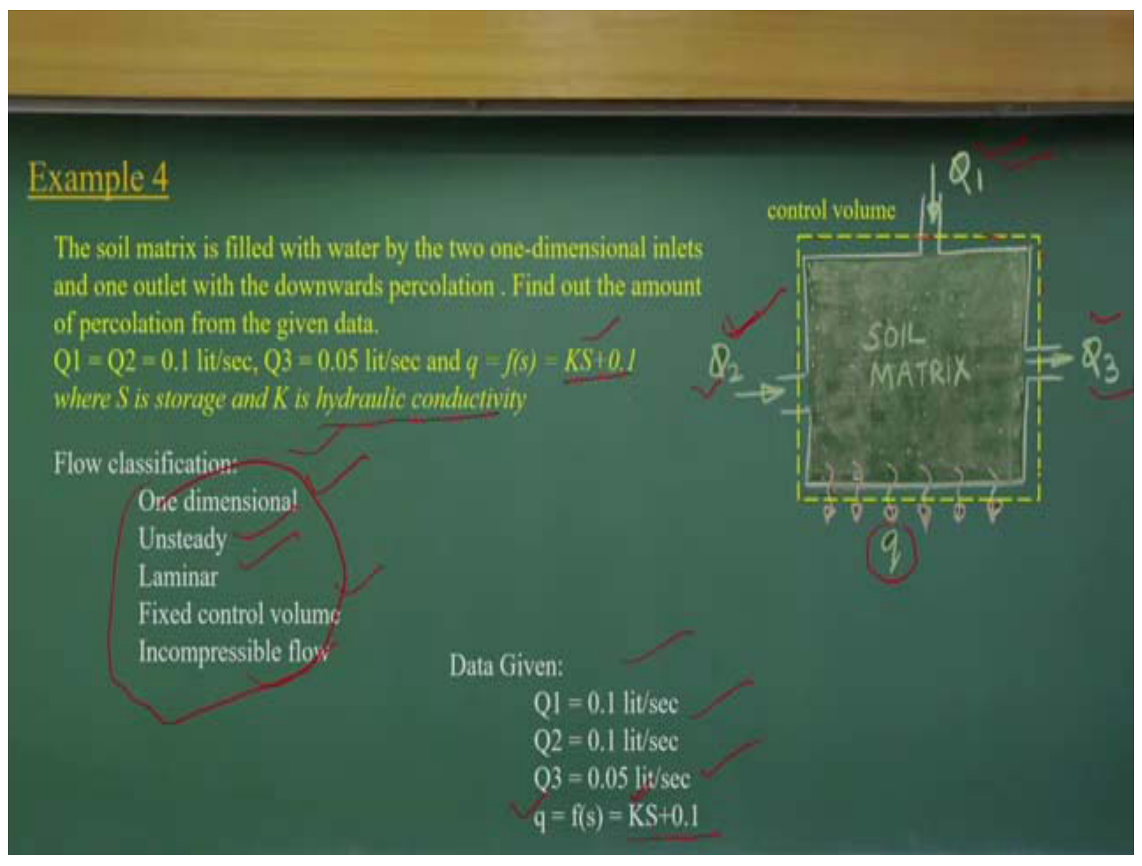

For this, let us come back to the example. Last time we discussed this problem. Again I am going to repeat it just for more detailed understanding of these problems. Let us consider that there is a soil matrix, that means there are soils that are there which is having porous space, and in that soil component we have the flow. The water is coming, it is Q1, Q2, and Q3 is going out. And at the bottom, there is percolation or seepaging.

[The soil matrix is filled with water by the two one-dimensional inlets and one outlet with the downwards percolation. Find out the amount of percolation from the given data.

Q1 = Q2 = 0.1 lit/sec, Q3 = 0.05 lit/sec and $q = f(s) = KS+0.1$

where S is storage and K is hydraulic conductivity]

The water is coming out from the soil matrix. In the porous space of the soil water is there, that is what is coming out as seepage water to here. Here, this q is a function of storage within the soil matrix and the K is a constant proportional or we can call hydraulic conductivity. So, Q1 is given for this study, Q2 is given, Q3 is given. So, I have taken this is my control volume. If you look at the yellow colors it is the control volume.

If that is the control volume, before applying this conservation of mass I should classify the problem. The problem is what nature, it is one dimensional flow. The flow what we can consider across this control surface is one-dimensional.

Flow classification:

- One dimensional
- Unsteady
- Laminar

Fixed control volume

Incompressible flow

Data Given:

Q1 = 0.1 lit/sec

Q2 = 0.1 lit/sec

Q3 = 0.05 lit/sec

q = f(s) = KS+0.1

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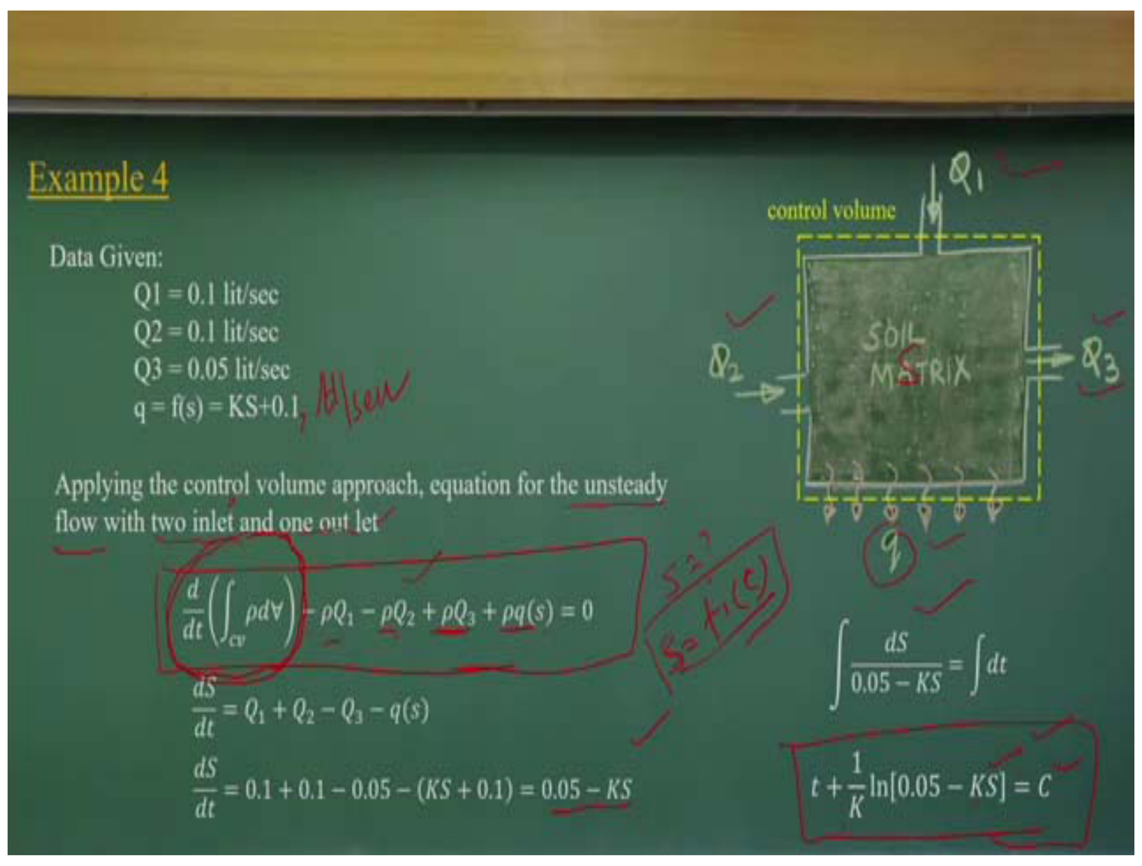

Now, I have to simplify the problem. I have to apply under this control volume the basic mass conservation equations. It is unsteady equation with two inlets and one outlet. That is what you can do, Q1 and Q2 are inlets, Q3 is outlet. Then, the outlet is a seepage part which is going out which will be in terms of S. If you look, when I consider this is the control volume having Q1, Q2, the inflows, and Q3 is outflow, and q is also outflow, which is a functions with respect to S, the storage.

Applying the control volume approach, equation for the unsteady flow with two inlet and one out let

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) - \rho Q_1 - \rho Q_2 + \rho Q_3 + \rho q(s) = 0$$

The control volume's K is 0.1 which unit will be litre per second. Then, I just apply the unsteady flow equations of convergence of mass. This is the volume integrals part if you remember it, and since it is a one-dimensional part we have the negative for the inflows and positive for the outflows. So, you can find out this Q3s and all. All are the mass flow in, mass flow out, and that is the integration. And this part is the storage, S.

$$\frac{dS}{dt} = Q_1 + Q_2 - Q_3 - q(s)$$

$$\frac{dS}{dt} = 0.1 + 0.1 - 0.05 - (KS + 0.1) = 0.05 - KS$$

So, dS by dt, Q1 plus Q2, and this is just rearrangement of this and substituting this value we will get the dS dt is these functions. And we can integrate it and finally get a relationship between d and S as K is the constant, is equal to C. So, if you have the boundary conditions we can determine the C value, then we can know what is the function of S, how the S varies with respect to the time. That is our problem.

$$\int \frac{dS}{0.05 - KS} = \int dt$$

$$t + \frac{1}{K}\ln[0.05 - KS] = C$$

So, that way, if you look, very complex problems like this, when you have a soil matrix and porous structure and you have the flow of Q1, Q2 inflows, and outflow is there, the seepage is a function of how of water storage within the soil matrix. We can apply a simple mass conservation equation for this control volume. Then we can integrate it to get what is the function of S with respect time and that is what will give us from this. So, this is about the problem. Again, I solved it for you.

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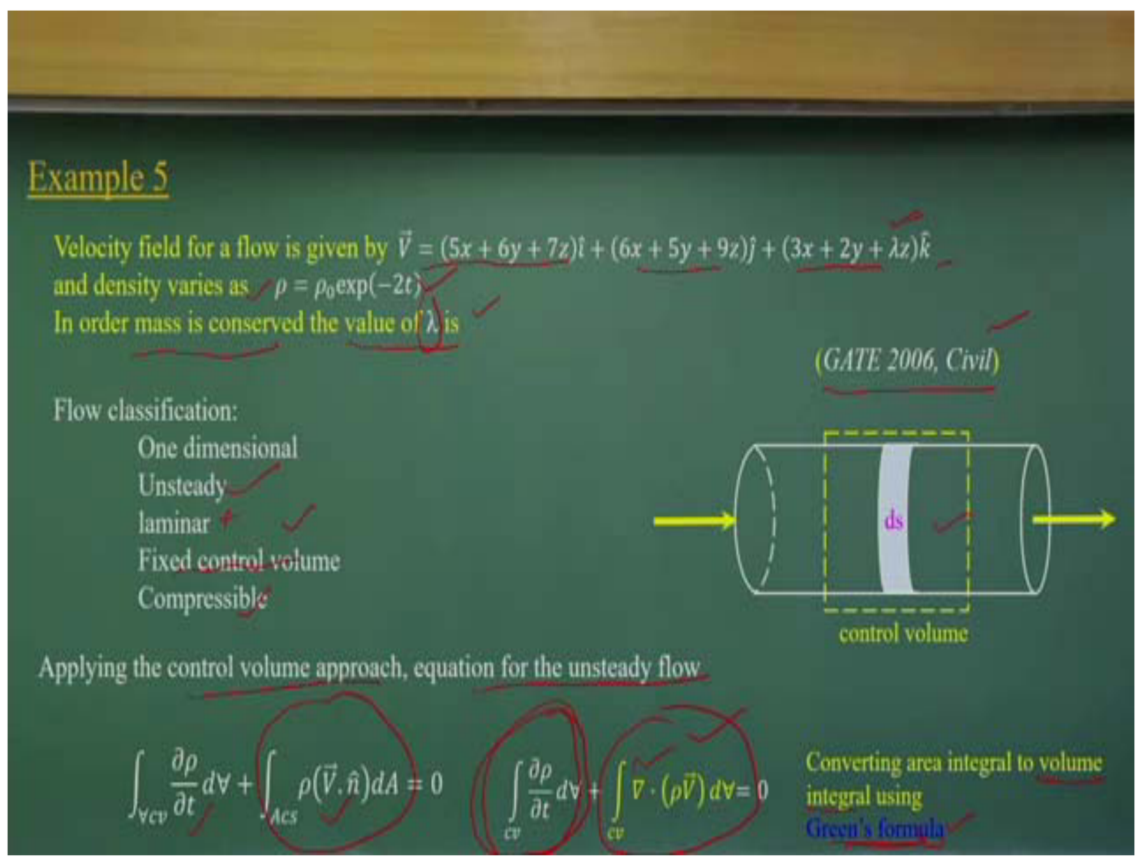

So, now, let us come to example five which is the GATE 2006 civil engineering part. In that problem the velocity field is given. If you see this, the scalar component of the velocity field in which there is a component of lambda is unknown to us, okay? Density also varies with respect to the time. That means it is unsteady problems. And the velocity fields are given.

[Velocity field for a flow is given by $\vec{V} = (5x + 6y + 7z)\hat{\imath} + (6x + 5y + 9z)\hat{\jmath} + (3x + 2y + \lambda z)\hat{k}$ and density varies as $\rho = \rho_0 \exp(-2t)$

In order mass is conserved the value of λ is

(*GATE 2006, Civil*)]

And here that mass is conserved, the point is that mass is conserved, then what could be the value of lambda. That is what will be different. So, that means what we will do is we will apply the mass conservation equations and once that mass conservation equation is satisfied, from that mass conservation equation we will compute what will be the lambda value. That is the problem here. So, let me classify the problem.

Flow classification:

- One dimensional
- Unsteady
- laminar
- Fixed control volume
- Compressible

That means it is some sort of the volume like this, you have a dS like this. So, applying this control volume approach, equations for unsteady flow, you will have this component and this component which already we derived earlier. Let me put it this form, okay?

Applying the control volume approach, equation for the unsteady flow

$$\int_{\forall cv} \frac{\partial \rho}{\partial t} d\forall + \int_{Acs} \rho\left(\vec{V}.\hat{n}\right) dA = 0$$

$$\int_{cv} \frac{\partial \rho}{\partial t} d\forall + \int_{cv} \nabla \cdot \left(\rho \vec{V}\right) d\forall = 0$$

(Converting area integral to volume integral using - Green's formula)

The time derivative part of the control volumes that is what we will have this part. See, if you look this surface integrals, if I follow this Green's formula, these surface integrals can be converted to volume integrals in terms of delta operators, okay? If you remember this Green's formula, we can convert this surface integral into the control volume levels having delta dot products. That is the concept we could have known from the mathematics point of view.

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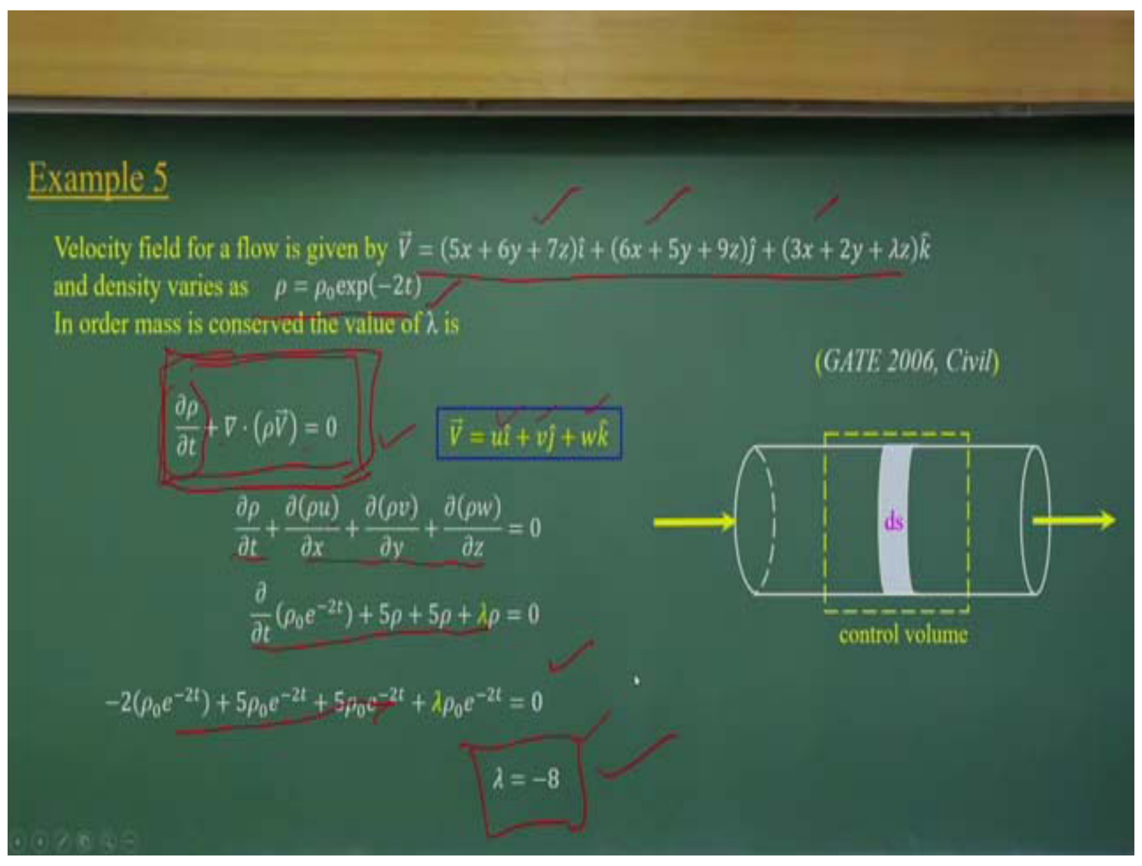

Now, if I have that part and if you do the integral part, I can go out.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right) = 0$$

Then, finally, the equation becomes this because when you do integration over that, that is common, we can take it out.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{\partial}{\partial t}(\rho_0 e^{-2t}) + 5\rho + 5\rho + \lambda\rho = 0$$

$$-2(\rho_0 e^{-2t}) + 5\rho_0 e^{-2t} + 5\rho_0 e^{-2t} + \lambda\rho_0 e^{-2t} = 0$$

$$\lambda = -8$$

Why, this u, v, w are scalar component, you can get this component as this one, okay, Finally, I get these relations here, and this relation will finally give me the lambda value which is equal to minus 8. So, this is the problem that we solved. So, basically, this equation is conservation of mass, what we have applied.

But since the velocity vectors are there which is having three dimensional velocity vectors and the density is a function of the time, so, we have applied it as bringing to this level. Then, we have solved for the lambda value.

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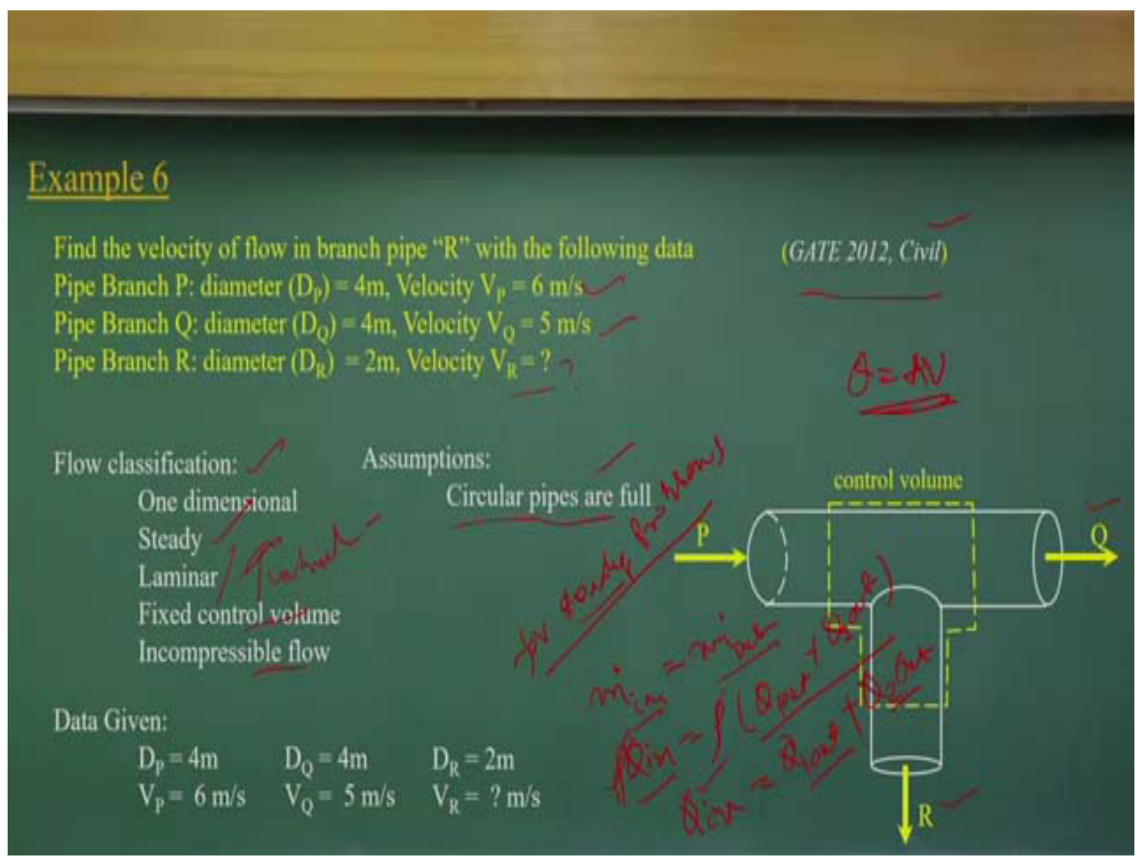

Now, we take another example which is given in GATE 2012 civil engineering specialization. There it is a very simple problem, like what is given in this diagram. The pipe is there and there is a joint which is called a T-joint like this. P is inflow that is coming. Q is going out from this. R is going from this out. The pipes is having branching of P, Q, R. The diameters are given. The velocities V_P and V_Q are given. V_R to be estimated which is very simplified problem. You can see that this problem is one-dimensional, steady.

[Find the velocity of flow in branch pipe "R" with the following data

Pipe Branch P: diameter (D_P) = 4m, Velocity V_P = 6 m/s

Pipe Branch Q: diameter (D_Q) = 4m, Velocity V_Q = 5 m/s

Pipe Branch R: diameter (D_R) = 2m, Velocity V_R = ?]

Flow classification:

- One dimensional
- Steady
- Laminar
- Fixed control volume
- Incompressible flow

Assumptions:

- Circular pipes are full

Velocity is given. The circular pipes are full. The flow could be laminar or turbulent, okay? We do not know it. Fixed control volume and incompressible. So, this is a very simple problem. Looking at this we will just apply the control volume and try to find out what will be the mass. Inflow is coming into this and going out. Basically, if you try to remember mass in plus mass per unit time coming in should be equal to mass inflow from going out for steady problems. So, being a steady problem, what we have.

Data Given:

$D_P = 4m$

$V_P = 6$ m/s

$D_Q = 4m$

$V_Q = 5$ m/s

$D_R = 2m$

$V_R =$? m/s

The mass inflow what is coming, rate of mass inflow is what is coming in, it should be equal to rate of mass inflow going out from this control volume, that is the thing. So, in this case, because there are two outlets, sum of this two masses outflows going out from this, that is equal to mass inflow that is coming in, mass flow rate that is coming in, the mass per unit time that is coming in.

So, that way you can see that if you have ρ Q in, a very simple, ρ Q in will be rho Q1 out plus Q2 out. That is the basic concept. Since it is same density, that means you have Q in is equal to Q1 out or Q2 out to outlet. So, sum of the two volumetric discharge is equal to the inflow volumetric discharge what is going. That is very simple problem. Only, you have to compute. Since the velocity is given, so Q will be Q into V.

That is the basic concept, area into velocity is Q, average velocity is given to us, so we can compute the discharge and we just applied the Q in is equal to Q1 out plus Q2 out.

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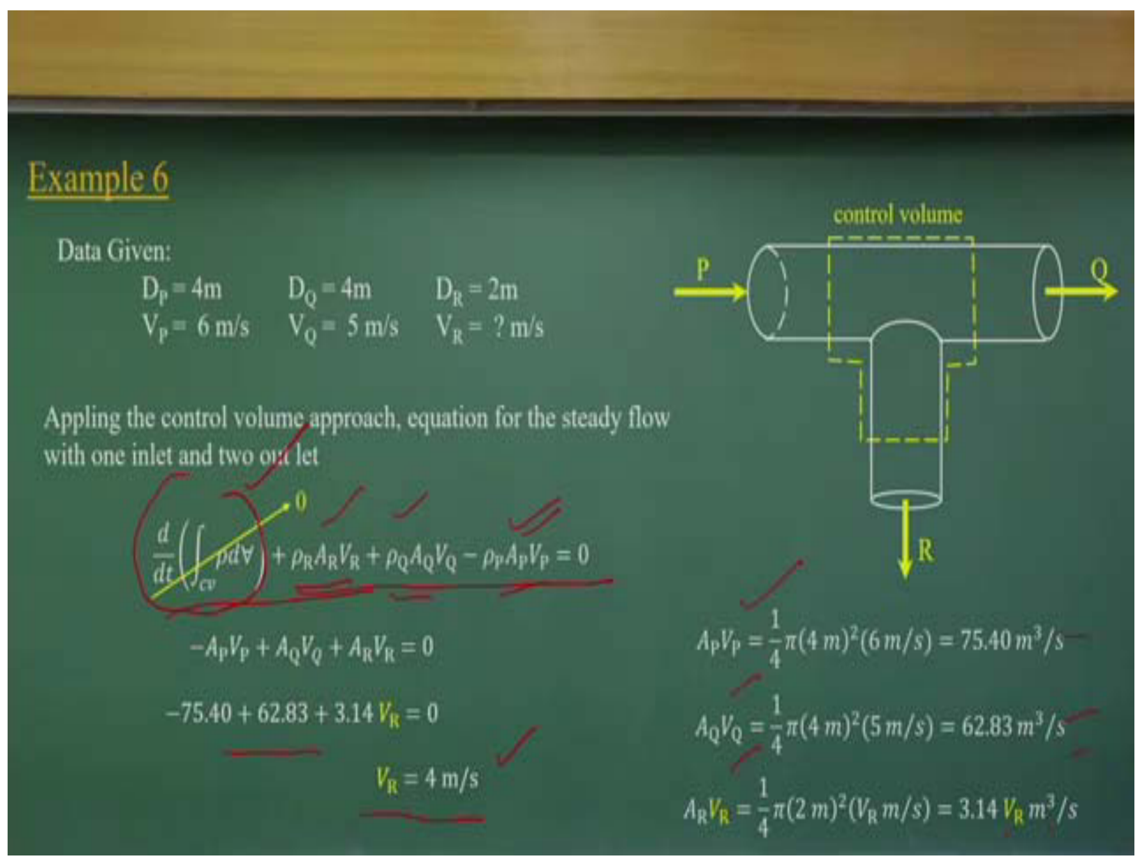

So, numerically that is what is coming. For the steady flow this becomes zero. So, you have in and out. As you know, this in will be negative and both out will be positives, and substituting these Q values for all the cases, with V R unknown. So finally, substituting to this equations will give V R equal to 4 m/s. So, very simple form of solving the pipe problems where you have one inlet and two outlets.

Appling the control volume approach, equation for the steady flow with one inlet and two out let

0

$$\frac{d}{dt}\left(\int_{cv} \rho d\forall\right) + \rho_{\mathrm{R}} A_{\mathrm{R}} V_{\mathrm{R}} + \rho_{\mathrm{Q}} A_{\mathrm{Q}} V_{\mathrm{Q}} - \rho_{\mathrm{P}} A_{\mathrm{P}} V_{\mathrm{P}} = 0$$

$$-A_{\mathrm{P}} V_{\mathrm{P}} + A_{\mathrm{Q}} V_{Q} + A_{\mathrm{R}} V_{\mathrm{R}} = 0$$

$$A_{\mathrm{P}} V_{\mathrm{P}} = \frac{1}{4}\pi(4\,m)^2(6\,m/s) = 75.40\,m^3/s$$

$$A_{\mathrm{Q}} V_{\mathrm{Q}} = \frac{1}{4}\pi(4\,m)^2(5\,m/s) = 62.83\,m^3/s$$

$$A_{\mathrm{R}} V_{\mathrm{R}} = \frac{1}{4}\pi(2\,m)^2(V_{\mathrm{R}}\,m/s) = 3.14\,V_{\mathrm{R}}\,m^3/s$$

As it is incompressible flow, the density is constant, so you just do volumetric flux coming in is equal to the sum of the volumetric flux going out from the control volume. That is what we commit. If you do not remember that, very simple way you remember it is that the mass influx or rate of change of mass with respect to time coming into the control volume should be equal to the rate of the mass going out from the control volume, that should be equal.

$-A_{\mathrm{P}} V_{\mathrm{P}} + A_{\mathrm{Q}} V_{Q} + A_{\mathrm{R}} V_{\mathrm{R}} = 0$

$$-75.40 + 62.83 + 3.14\, V_R = 0$$
$$V_R = 4 \text{ m/s}$$

Mass influx and outflux rate should be equal. That is the concept if we consider for steady flow conditions.

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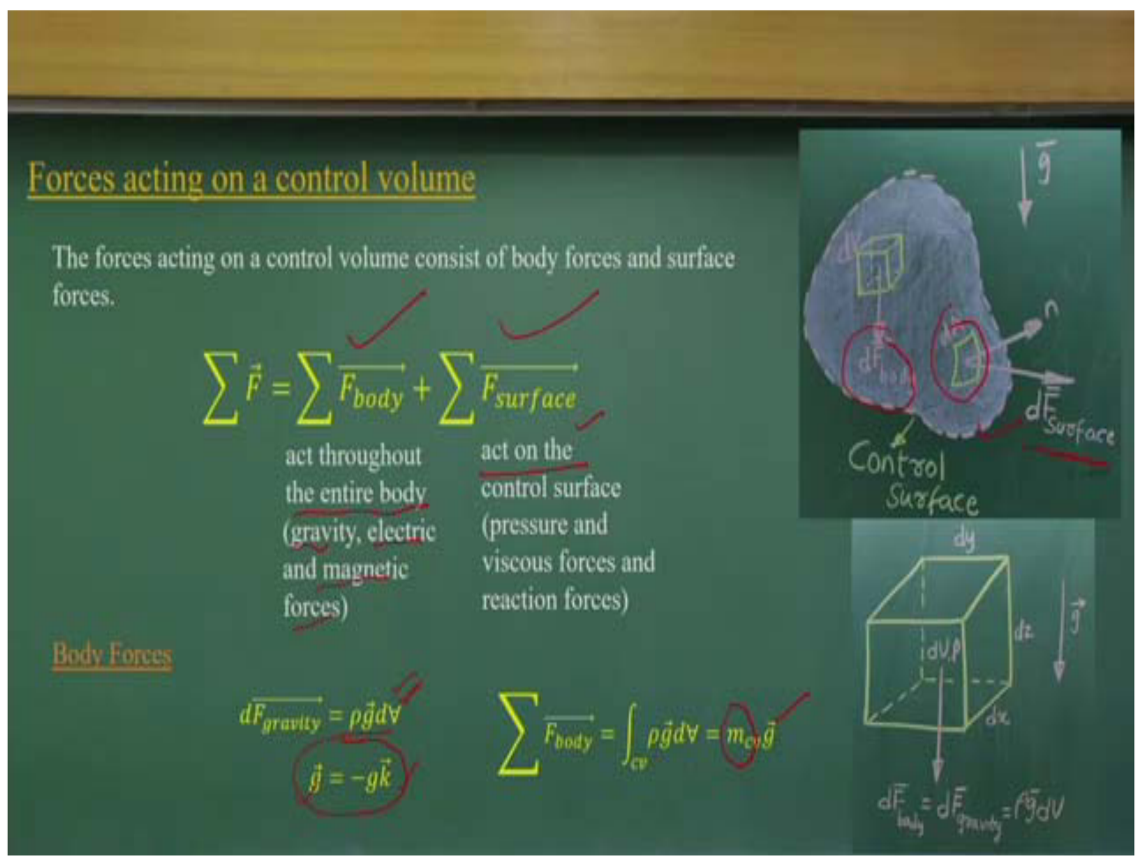

Now, let us come to derive the linear momentum equations, okay? So, we are going for solving these flow problems for linear momentum equations. That means we will consider the control volumes, we have considered the control volume like this. So, each control volume has the control surface. It could be a very simple tetrahedral type of structure or you can have very complex, it does not matter what could be the shapes, okay?

It can have simple shapes or it can have very complex shapes. So, if you look at that, over that surface what will happen is you will have normal vectors, let dA be the surface area, over that is the normal vector to the surface area. So, you will have the surface which will have two types forces going to act on this. One is the body force, that because of the mass of the control volume, how much of body force is giving, say, gravity point of view, or other forces we do not consider here is electrical or magnetic field point of view.

So, basically, because of the control volume mass, how much of weight you are getting, how much of body force you are getting, that is what will be the body force. And along the surface the forces that is acting that is surface force. So, you have two force components you get for a control volume. One is the body force and the other is surface force. The body force acting in the entire body, that is what is most often the gravity.

The other electric and magnetic fields are not considered for this case, but some cases we can consider it. The other is the surface force acting on the control surface. So, the forces acting on the control surface will be the pressure force, the viscous force, and any reaction forces, okay. Like a control surface is cutting through a surface, a rigid part. So, there could be a reaction force that will be there.

$$\sum \vec{F} = \sum \overrightarrow{F_{body}} + \sum \overrightarrow{F_{surface}}$$

So, that way if you look, if we take a control volume there results to be two types of forces. One is the body force which acts throughout the entire body, that depends on the mass present within the control volume. Another is that over the surface of control volume there will be the force components, those forces are due to pressure, viscous forces because of viscosities of the fluid flow systems, there will be the viscous force component, and also reaction forces or the other force component comes in.

Now, let me find out what will be the gravity force, which is a very easy thing. If I take a small element dV, I will have the weight of these small control volumes, it will be ρ, g, and dV. So, look at the unit of each component, if you can understand that. ρdV will be the mass, d V is here. Look for the volume. Mass into g is the gravity force component. Here, the gravity force component, we can consider g is a vector quantity of any direction.

$$d\overrightarrow{F_{gravity}} = \rho \vec{g} d\forall$$

$$\vec{g} = -g\vec{k}$$

But you can align with, if y is of direction, then the K notation we can use to define the g vector component, okay? So, basically, if I consider the total control volumes, then the sum of the force or volume integrals of this component ρ g d V that will be the gravity part or indirectly this is mass of the control volume into g, g is the acceleration due to gravity vector component. And many of the times we align the z-axis and g is downward, then we use the negative, not the scalar quantity, as a vector representation.

$$\sum \overrightarrow{F_{body}} = \int_{cv} \rho \vec{g} d\forall = m_{cv}\vec{g}$$

But if consider different orientation of the control volume, then you can consider g is a vector quantity, it has a scalar component of gx, gy, gz in three respective scalar direction of x, y, and z.

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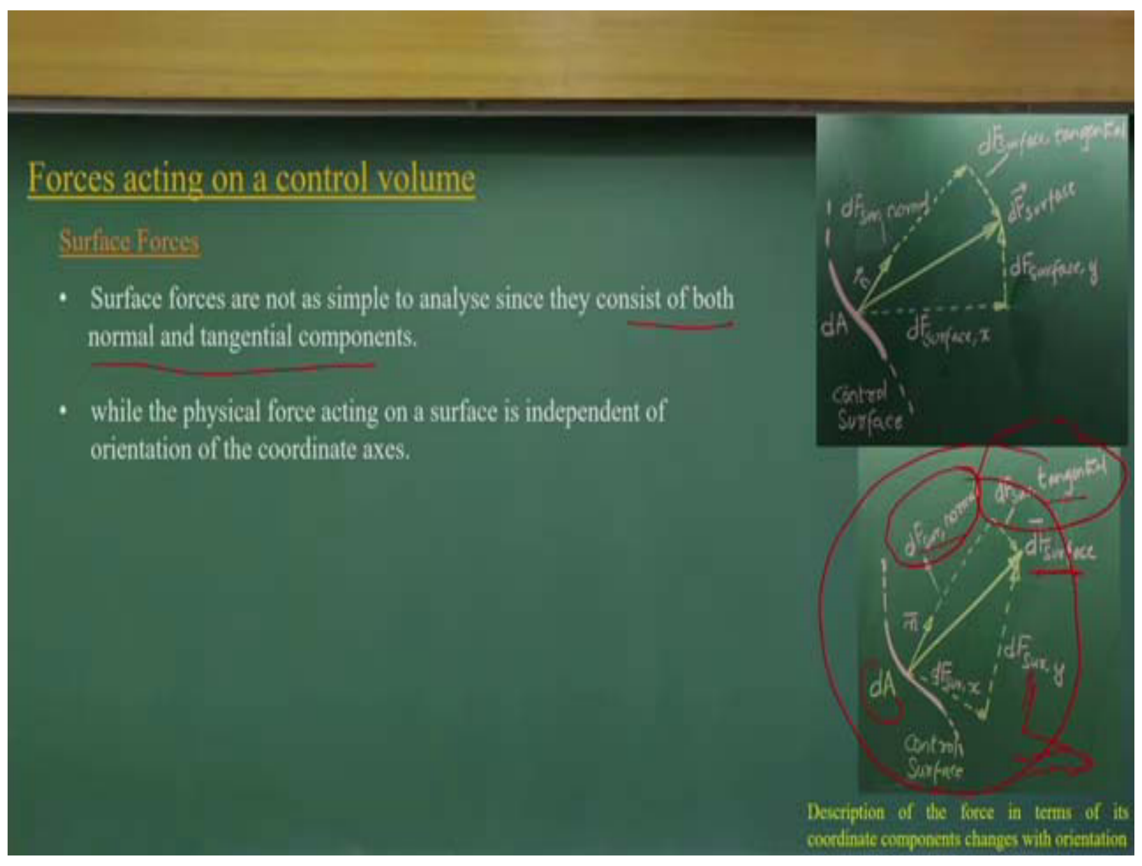

Now, let us come back to what type of force are acting. Surface forces as we discussed earlier will be there. Any surface force will have the normal component as well as the tangential component. Let us take this figure which is very interesting figure, showing to you. This is the control surface having the area of da. It is normal vector, n is this part, it is a normal vector.

So, if your force acting on this is having an angle, then this force can have two components. One is for the component for the normal, another is the tangential component, okay? So, the control surface can be considered in any orientation, okay? Over that you have a normal vector which is normal to the control surface there. So, if your force is acting at that point having a different angle, then what will happen is you will have a normal component and also the tangential component.

If you want the result as the x and y component, that is your Cartesian coordinates, that is what is different here. One is the Cartesian coordinate level resolving the force vector component to a scalar component in x and y direction. Another one is we are resolving this force component into the normal or the tangential component. That is what is illustrated here, how you can have two different components.

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